

Linear response conductance and magneto-resistance of ferromagnetic single-electron transistors

Arne Brataas

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138

X. H. Wang

Department of Physics, Lund University, Sölvegatan 14A, S-223 62 Lund, Sweden

(February 6, 2008)

The current through ferromagnetic single-electron transistors (SET's) is considered. Using path integrals the linear response conductance is formulated as a function of the tunnel conductance vs. quantum conductance and the temperature vs. Coulomb charging energy. The magneto-resistance of ferromagnet-normal metal-ferromagnet (F-N-F) SET's is almost independent of the Coulomb charging energy and is only reduced when the transport dwell time is longer than the spin-flip relaxation time. In all-ferromagnetic (F-F-F) SET's with negligible spin-flip relaxation time the magneto-resistance is calculated analytically at high temperatures and numerically at low temperatures. The F-F-F magneto-resistance is enhanced by higher order tunneling processes at low temperatures in the 'off' state when the induced charges vanishes. In contrast, in the 'on' state near resonance the magneto-resistance ratio is a non-monotonic function of the inverse temperature.

I. INTRODUCTION

The spin of the electron can provide new functionality in electronic devices¹ and spin-polarized transport is therefore an active research field. Spin-transport was pioneered by Tedrow and Meservey² and the discovery of the giant magnetoresistance effect (GMR) in metallic multilayers has motivated a substantial re-newed interest in this field. Recently spin-polarized current injection in all-semiconductor devices³ and carbon nanotubes⁴ have been realized. The fabrication of tunnel junctions made of two ferromagnetic leads separated by an insulating layer has become well under control.^{5,6} These technological advancements can make it possible to design new generation of electronic devices such as magnetic RAM's, sensors and ultimately perhaps quantum computers.

In this article, we will focus on an elementary structure, the ferromagnetic single-electron transistor and discuss the role of the ferromagnetic electrodes, spin-accumulation and Coulomb charging effects on the transport properties. A schematic picture of the system under consideration is shown in Fig. 1. A normal (ferromagnetic) metal island is coupled to two ferromagnetic reservoirs by tunnel junctions. There is a capacitive coupling C between the metal island and the metal reservoirs. The

island is so small so that the charging energy $E_c = e^2/2C$ associated with the addition of a single electron to the island can be larger than both the temperature and the applied source-drain bias. The electrostatic potential on the island can be controlled by a gate voltage V_g capacitively coupled via a capacitance C_g to the island. We consider an island weakly coupled to the reservoirs by many channels with transmission probabilities much less than unity, but the total tunnel conductance G can be smaller or larger than the quantum conductance G_K . In the simplest case, the single-electron transistor (SET) comprising of tunnel junctions with junction resistances much larger than the quantum resistance $R_K = h/e^2$ exhibits Coulomb blockade of the single electron tunneling at low temperatures when the gate voltage vanishes, and a conductance peak when $C_g V_g = e/2$. For spin-polarized charge transfer the different transport properties of the spin-up and spin-down electrons have to be taken into account. Therefore the behavior of the SET is determined by the interplay between spin-dependent transport and Coulomb charging effects.⁷⁻¹³

All-ferromagnetic (F-F-F) SET's have recently been investigated experimentally⁶ and theoretically.^{7,8,13} The tunneling magneto-resistance (TMR) in F-F-F systems is defined as the relative conductance difference by changing the magnetization of the ferromagnetic island from being parallel to anti-parallel to the magnetizations of the ferromagnetic reservoirs (which are kept parallel). Ferromagnetic islands have a short spin-flip relaxation time and several works have computed the transport properties of F-F-F systems disregarding spin-accumulation.^{7,8,13} An enhanced TMR in the Coulomb blockade regime was found experimentally,⁶ and it was subsequently shown that the TMR could be doubled due to co-tunneling⁸ and that higher order tunneling processes could give even larger enhancements of the TMR.¹³ In what follows, we will use a non-perturbative approach to calculate the tunneling magneto-resistance (TMR) of the F-F-F SET showing that even larger enhancements of the TMR than what has been reported in Ref. 13 is possible. We will also discuss the TMR in the 'on'-state when the Coulomb blockade effect is lifted. It will be demonstrated that the TMR in this regime is a non-monotonic function of the temperature.

The physics of the TMR in F-N-F systems is completely different from the mechanism of the TMR in F-

F-F systems. F-F-F systems exhibit a finite TMR even in the absence of any spin-accumulation on the ferromagnetic island. The transport properties can be modeled by an all-normal metal device with magnetization configuration dependent tunnel conductances. In contrast, the TMR of F-N-F systems is uniquely related to the spin-accumulation on the island. Spin-accumulation governs the transport properties and leads to a measurable TMR when the spin-flip relaxation time τ_{sf} is longer than the dwell time so that¹⁰

$$\tau_{sf}/(\chi h) > G_K/G, \quad (1)$$

where χ is the spin-susceptibility, G is the tunnel conductance and $G_K = e^2/h$ is the quantum conductance. In order to realize spin-accumulation a small spin-susceptibility is required (1) which implies a small normal metal island with a large Coulomb charging energy E_c and consequently Coulomb blockade effects should be taken into account.¹⁰ When the tunnel conductance G is much smaller than the quantum conductance G_K and the temperature is not too low, transport can be described by sequential tunneling processes and this was performed in Refs. 9–11. However, Eq. (1) shows that spin-accumulation is easier to realize when the tunnel conductance is large, $G > G_K$. In this strong tunneling limit the quantum fluctuations of the charge on the island as well as the spin-accumulation should be taken into account with a theoretical description that goes beyond the low order perturbative treatment presented in Refs. 9–12. The tunneling magneto-resistance (TMR) in F-N-F systems is defined in a different way than for (F-F-F) systems: The TMR is the relative conductance difference on going from a configuration where the magnetizations in the ferromagnetic reservoirs are parallel to a configuration where the magnetizations are anti-parallel $(g_P - g_{AP})/g_P$. Changing the magnetization configuration in F-N-F systems only changes the spin-dependence of the tunnel conductances but conserves the total tunnel conductances. Hence, a much weaker coupling to the charge degrees of freedom of the island and a reduced influence of Coulomb charging should be expected. In the Coulomb blockade regime at low temperatures the total conductance through the system is suppressed. Consequently, also the transport of spins into the island decreases and at sufficiently low temperatures the dwell time is longer than the spin-flip relaxation time so that (1) is no longer satisfied. The spin-accumulation and thus the TMR must therefore be reduced in the Coulomb blockade regime which is contrary to the behavior of the TMR in F-F-F systems. We will show that the linear response TMR in F-N-F systems in the Coulomb blockade regime equals or is smaller than the classical high-temperature TMR, which is in contrast to the results for F-F-F systems.

We present in this paper a unified analytical expression for the linear response conductance of F-F-F and F-N-F SET's with dirty metallic islands. Our formula is

valid to all orders of the ratio between the tunnel conductances and the quantum conductance, for arbitrary spin-flip relaxation times and takes into account general band-structures for the ferromagnetic reservoirs and the normal/ferromagnetic metal island. Our results are in this sense general. The analytical expression for the linear response conductance is used to discuss the magneto-resistance of F-F-F and F-N-F SET's.

The paper is organized in the following way. The system is described and the model Hamiltonian introduced in Section II. In Section III the linear response conductance is calculated, including the multi-band effect of the electrodes and the spin-accumulation on the central island. The TMR is evaluated in Section IV in the case of ferromagnet-normal metal-ferromagnet SET's and in Section V in the case of all-ferromagnet SET's. Section VI concludes the paper.

II. MODEL

We consider a ferromagnetic single-electron transistor comprising of a normal/ferromagnetic metal island connected to ferromagnetic leads by tunnel junctions. The central island is capacitively coupled to a gate voltage V_g via a capacitance C_g . The Hamiltonian of such a device is

$$\hat{H} = \left(\hat{H}_l + \hat{H}_i + \hat{H}_r \right) + \hat{H}_c + \left(\hat{H}_{li} + \hat{H}_{ri} + \text{h.c.} \right). \quad (2)$$

The quasi-particles in the left reservoir are described by

$$\hat{H}_l = \sum_{\mathbf{l} \sigma n m} W_{\mathbf{l} \sigma}^{nm} \hat{c}_{\mathbf{l} \sigma}^{n\dagger} \hat{c}_{\mathbf{l} \sigma}^m, \quad (3)$$

where n and m are the band-indices, σ is the electron spin, \mathbf{l} is the momentum in the left reservoir, $W_{\mathbf{l} \sigma}^{nm}$ determine the quasi-particle energy bands and the hybridization between the bands for a given spin, and $\hat{c}_{\mathbf{l} \sigma}^{n\dagger}$ creates an electron with spin σ and momentum \mathbf{l} in band n in the left reservoir. The hat ($\hat{}$) denotes an operator. There are similar Hamiltonians for the quasi-particles on the island ($\mathbf{l} \rightarrow \mathbf{i}$) and the quasi-particles in the right reservoir ($\mathbf{l} \rightarrow \mathbf{r}$). The Coulomb charging effects are included via

$$\hat{H}_c = \frac{e^2}{2C} \left(\sum_{\sigma} \hat{N}_{\sigma}^i - n_{\text{ex}} \right)^2, \quad (4)$$

where the induced electron number is $n_{\text{ex}} = C_g V_g / e$ and the spin-dependent number of excess electrons on the island is $\hat{N}_{\sigma}^i = \sum_{\mathbf{i} n} \hat{c}_{\mathbf{i} \sigma}^{n\dagger} \hat{c}_{\mathbf{i} \sigma}^n$. We will below also make use of the spin-dependent excess number of electrons in the left (right) reservoir, $\hat{N}_{\sigma}^l = \sum_{\mathbf{l} n} \hat{c}_{\mathbf{l} \sigma}^{n\dagger} \hat{c}_{\mathbf{l} \sigma}^n$ ($\hat{N}_{\sigma}^r = \sum_{\mathbf{r} n} \hat{c}_{\mathbf{r} \sigma}^{n\dagger} \hat{c}_{\mathbf{r} \sigma}^n$). The tunneling between the reservoirs and the island is taken into account via the tunneling Hamiltonians $\hat{H}_{pi} = \sum_{\sigma} \hat{H}_{pi\sigma}$ ($p = l$ denotes left and $p = r$ denotes right):

$$\hat{H}_{pi\sigma} = \sum_{\mathbf{pinm}} t_{\mathbf{pi}\sigma}^{nm} \hat{c}_{\mathbf{p}\sigma}^{n\dagger} \hat{c}_{\mathbf{i}\sigma}^m. \quad (5)$$

The tunneling matrix elements $t_{\mathbf{pi}\sigma}^{nm}$ allow tunneling between different bands with the same spin. Spin-flip scatterings during the tunneling processes are disregarded, but can be included in the formalism leading to a renormalization of the spin-asymmetries of the junction resistances.

We consider the situation when the temperature is much larger than the level spacing and the level spacing is much smaller than the Coulomb charging energy. The level spacing is consequently taken to be continuous. Co-tunneling and higher order tunneling processes have both elastic and inelastic contributions,¹⁷ e.g. in the case of co-tunneling the elastic contribution means that the same electron tunnels through both of the two junctions whereas the inelastic contribution corresponds to two different electrons that tunnel in the two junctions: One tunnels into the island above its Fermi level, and another jumps out of the island from below the Fermi level. The elastic contribution is sensitive to the phase of the electron on the island and consequently becomes vanishingly small for dirty systems when the characteristic time of tunneling through the macroscopic barrier caused by the Coulomb energy \hbar/E_c ($E_c = e^2/(2C)$) is larger than the classical time L^2/D for diffusion through the island (L is the size of the island and D is the diffusion coefficient).¹⁷ We consider the inelastic transport regime exclusively and disregard the elastic contributions to the tunneling processes, i.e. for an estimate of a diffusion constant on the island of $D = 10\text{cm}^2/\text{s}$ and a charging energy of $E_c = 1\text{K}$ we assume that the size of the island $L < 0.1\mu\text{m}$ (in reality there is also scattering at the boundary of the island that further relax this assumption). The occupation of the energy levels on the island can then be described by an energy ϵ and spin σ dependent non-equilibrium distribution $f_{\sigma}^i(\epsilon)$. The non-equilibrium distribution $f_{\sigma}^i(\epsilon)$ is spin-dependent to allow a non-equilibrium spin-accumulation and only equals the equilibrium Fermi-Dirac distribution $f(\epsilon - \mu)$ when the bias voltage vanishes (μ is the equilibrium chemical potential). Furthermore, the leads and the island are metallic so that the tunnel conductances (see Eq. (16) below) are energy independent on the scale of the Coulomb charging energy which is much smaller than the Fermi energy. We consider the linear response regime where the bias voltage is the smallest energy scale in the system. Under these assumptions, only the total occupation of spin-up and spin-down electrons on the island, $\int d\epsilon [f_{\sigma}^i - f(\epsilon - \mu)]$, determine the macroscopic Coulomb barrier and the total effective tunneling rates. Consequently to all orders in perturbation theory of the tunneling Hamiltonian, we can describe the electron occupation on the island by a local chemical potential $\mu_{\sigma}^i \equiv \mu + \int d\epsilon [f_{\sigma}^i(\epsilon) - f(\epsilon - \mu)]$ since the energy-dependence of the distribution $f_{\sigma}^i(\epsilon)$ does not influence the transport properties. It is thus e.g. irrelevant if there is substantial energy-relaxation on the

island so that the non-equilibrium distribution $f_{\sigma}^i(\epsilon)$ approaches the Fermi-Dirac distribution $f(\epsilon - \mu_{\sigma}^i)$ or if there is no energy-relaxation so that the non-equilibrium distribution $f_{\sigma}^i(\epsilon)$ is a linear combination of the Fermi-Dirac distributions in the left and right reservoirs with different local chemical potentials.

The electrons in the right reservoir and in the left reservoir are in local equilibrium with chemical potentials $\mu^l(t)$ and $\mu^r(t)$, respectively and as seen above the electrons on the island can be described as in local equilibrium with a spin-dependent chemical potential $\mu_{\sigma}^i(t)$. The temperature is the same in all subsystems. The thermal average of the spin-dependent current from the left ($p = l$) or right ($p = r$) reservoir to the island can then be written as

$$I_{\sigma}^p(t) = ie\text{Tr} \left[\left(\hat{H}_{pi\sigma}(t) - \hat{H}_{pi\sigma}^{\dagger}(t) \right) \exp(-\beta K(t)) \right] \quad (6)$$

with the non-equilibrium time-dependent grand canonical potential

$$\hat{K}(t) = \hat{H} - \sum_{\sigma} [\mu^l(t) \hat{N}_{\sigma}^l + \mu^r(t) \hat{N}_{\sigma}^r + \mu_{\sigma}^i(t) \hat{N}_{\sigma}^i]. \quad (7)$$

The spin-dependent chemical potentials on the island μ_{\uparrow}^i and μ_{\downarrow}^i should be determined from the flux of particles and spins into the island. The spin-flux into the island is¹⁰

$$(I_{\uparrow}^l + I_{\uparrow}^r) - (I_{\downarrow}^l + I_{\downarrow}^r) = \frac{es}{\tau_{\text{sf}}}. \quad (8)$$

Current conservation through the system requires

$$(I_{\uparrow}^l + I_{\uparrow}^r) + (I_{\downarrow}^l + I_{\downarrow}^r) = 0. \quad (9)$$

The excess number of spins on the island s is related to the non-equilibrium difference in the chemical potentials by the spin-susceptibility χ_s via $s = \chi_s(\mu_{\uparrow}^i - \mu_{\downarrow}^i)$.^{10,14} For non-interacting electrons $\chi_s = D$, where D is the density of states. The expression for the current (6) and the conservation laws for spins (8) and particles (9) uniquely determine the spin-dependent local chemical potentials on the island, $\mu_{i\uparrow}$ and $\mu_{i\downarrow}$, and consequently the current through the device. We will focus on the linear response regime in what follows.

III. LINEAR RESPONSE CONDUCTANCE

In the linear response regime a perturbation expansion in terms of the small differences in the chemical potentials on the left reservoir, on the island and on the right reservoir can be performed: $\mu^l(t) = \mu + \delta\mu^l(t)$, $\mu^r(t) = \mu + \delta\mu^r(t)$, and $\mu_{\sigma}^i(t) = \mu + \delta\mu_{\sigma}^i(t)$. The non-equilibrium grand canonical potential is $\hat{K}(t) = \hat{K}_0 + \delta\hat{K}(t)$, where the time-independent unperturbed grand canonical potential is $\hat{K}_0 = \hat{H} - \mu \sum_{\sigma} (\hat{N}_{\sigma}^l + \hat{N}_{\sigma}^r + \hat{N}_{\sigma}^i)$ and the time-dependent perturbation is $\delta\hat{K} = - \sum_{\sigma} (\delta\mu^l \hat{N}_{\sigma}^l +$

$\delta\mu^r \hat{N}_\sigma^r + \delta\mu_\sigma^i \hat{N}_\sigma^i$). We perform a time-dependent unitary transformation with the unitary matrix $\hat{U}(t) = \exp[i \int_{-\infty}^t dt_1 \delta\hat{K}(t_1)]$. The transformed grand canonical potential becomes $\hat{K}_t(t) = \hat{K}_0 + \delta\hat{H}(t)$, where to the lowest order in the non-equilibrium chemical potential differences

$$\delta\hat{H}(t) = \frac{1}{e} \sum_{qs} \hat{I}_s^q(t) \int_{-\infty}^t dt_1 [\delta\mu^q(t_1) - \delta\mu_s^i(t_1)] \quad (10)$$

and the sum over the parameter q corresponds to the left reservoir $q = l$ and the right reservoir $q = r$. The spin-currents through the left and the right barriers can be found from a standard linear response calculation:

$$I_\sigma^p(\omega) = \frac{i}{e\omega} \sum_{qs} \Pi_{\sigma s}^{pq}(\omega) [\delta\mu^q(\omega) - \delta\mu_s^i(\omega)], \quad (11)$$

where the retarded current-current correlation function is

$$\Pi_{\sigma s}^{pq}(t) = -i \left\langle \left[\hat{I}_\sigma^p(t), \hat{I}_s^q(0) \right] \right\rangle_{\text{eq}}, \quad (12)$$

This correlation function will be evaluated using path integrals. The computation is similar to the case of ultrasmall tunnel junctions or metallic SET that has been studied extensively.^{15,16} The main steps in the calculation of the correlation function is as follows.^{15,16} We use the equilibrium finite-temperature Matsubara formalism to find the retarded current-current correlation function. The imaginary time generating functional is defined as

$$Z[\{\eta\}] = \int D[\{c^*, c\}] D[\varphi] e^{-S_g}, \quad (13)$$

with the generalized action

$$S_g = - \int_0^\beta d\tau \text{Tr}_{\mathbf{p}n\sigma} c^*(\tau) \partial_\tau c(\tau) + \int_0^\beta d\tau K_0(\tau) - \int_0^\beta d\tau \sum_{p\sigma} I_\sigma^p(\tau) \eta_\sigma^p(\tau). \quad (14)$$

A Hubbard-Stratonovich transformation is performed to reduce the quadratic term in the charging energy to a linear coupling of the particle number operator with an auxiliary field.¹⁵ After integrating out the electronic degrees of freedom and including the principal contribution of the tunneling processes the generalized effective action is derived. The current-current correlation function can then be found by performing the functional derivative $\Pi_{\sigma s}^{pq}(\tau, \tau') = Z^{-1} \partial^2 Z / (\partial \eta_\sigma^p(\tau) \partial \eta_s^q(\tau'))|_{\eta \rightarrow 0}$. The resulting Fourier transform of the current-current correlation function is

$$\frac{1}{\omega} \Pi_{\sigma s}^{pq} = \frac{e^2}{h} [\delta_{p\sigma, qs} g_\sigma^p g_s^q + 2g_\sigma^p g_s^q g_1]. \quad (15)$$

The dimensionless tunnel conductance ($p = l, r$) is

$$g_\sigma^p = \sum_{jj'} D_{p\sigma}^j D_{i\sigma}^{j'} \left| \tilde{t}_{p\sigma}^{jj'} \right|^2, \quad (16)$$

Here $D_{i\sigma}^j$, $D_{r\sigma}^j$ and $D_{i\sigma}^j$ are the spin and band dependent densities of states and the renormalized transmission coefficient at the Fermi energy is $\tilde{t}_{p\sigma} = (U_{p\sigma})^\dagger t_{p\sigma} U_{i\sigma}$, where $U_{p\sigma}$ ($U_{i\sigma}$) is the material-specific matrix that diagonalizes the band Hamiltonian in reservoir p (island) for electrons with spin σ . In a single-band model the conductance is simply proportional to the density of states in the reservoir and the island, $g_\sigma^p = D_{p\sigma} D_{i\sigma} |t_\sigma|^2$. The relation is in general more complicated as shown in (16). The correlation function $g_0(i\omega_n)$ is

$$g_0 = \frac{4\pi}{i\omega_n} \int d\tau e^{i\omega_n \tau} \chi(\tau) \Gamma(\tau) \quad (17)$$

with the phase-phase correlation function

$$\Gamma(\tau) = \frac{1}{Z} \sum_{k=-\infty}^{\infty} e^{2\pi i k n_{\text{ex}}} \times \int_{b_k} D[\varphi] e^{-S[\varphi]} \cos[\varphi(\tau) - \varphi(0)]. \quad (18)$$

The effective action is

$$S[\varphi] = \int d\tau \frac{[\dot{\varphi}(\tau)]^2}{4E_c} - g^{\text{cl}} \int d\tau \int d\tau' \chi(\tau - \tau') \cos[\varphi(\tau) - \varphi(\tau')]. \quad (19)$$

with the boundary condition $b_k \rightarrow \varphi(\beta) = \varphi(0) + 2\pi k$. Since the leads and island are (ferro-)metallic with continuous spectra, the damping kernel $\chi(\tau)$ is an even function of the imaginary time τ with a period β and the Matsubara-Fourier components $\chi(\omega_l) = -|\omega_l|/4\pi$. The coupling of the island to the reservoirs is determined by the algebraic total dimensionless classical conductance $g^{\text{cl}} = \sum_{p\sigma} g_\sigma^p$. The partition function is $Z = \sum_k \int_{b_k} D[\varphi] \exp(-S[\varphi])$. Another correlation function in (15) is

$$g_1 = \frac{4\pi}{i\omega_n} \int d\tau e^{i\omega_n \tau} \frac{1}{Z} \times \sum_{k=-\infty}^{\infty} e^{2\pi i k n_{\text{ex}}} \int_{b_k} D[\varphi] e^{-S[\varphi]} I_t(\varphi, \tau) I_t(\varphi, 0) \quad (20)$$

where

$$I_t(\varphi, \sigma) = \int d\tau \chi(\sigma - \tau) \sin[\varphi(\sigma) - \varphi(\tau)]. \quad (21)$$

Due to the current conservation (9) the correlation function g_1 (20) does not appear in the final form of the stationary current.

Knowing the current-current correlation function, we can find the non-equilibrium chemical potential on the

island by using the conservation laws for spins (8) and particles (9). We find the spin-accumulation

$$\frac{\mu_{\uparrow}^i - \mu_{\downarrow}^i}{\mu^l - \mu^r} = \frac{(g_{\uparrow}^l g_{\downarrow}^r - g_{\downarrow}^l g_{\uparrow}^r)/(g_{\uparrow} g_{\downarrow})}{1 + g_{\text{sf}} [(g_0 g_{\uparrow})^{-1} + (g_0 g_{\downarrow})^{-1}]} \quad (22)$$

and the average chemical potential

$$\frac{\mu_{\uparrow}^i + \mu_{\downarrow}^i}{2} = \frac{g^l \mu^l + g^r \mu^r}{g^l + g^r} - \frac{g_{\uparrow} - g_{\downarrow}}{g_{\uparrow} + g_{\downarrow}} \frac{\mu_{\uparrow}^i - \mu_{\downarrow}^i}{2}, \quad (23)$$

where the spin-flip conductance g_{sf} is related to the spin-susceptibility χ_s and the spin-flip relaxation time τ_{sf} via¹⁰ $g_{\text{sf}} = \pi \chi_s / \tau_{\text{sf}}$. From the non-equilibrium chemical potentials on the island we obtain the current through the systems by using the relation for the current (11) and finally we calculate the conductance $g = eI/(\delta\mu^l - \delta\mu^r)$: The linear conductance in units of the quantum conductance e^2/h of the FSET is

$$g = \frac{g^r g^l}{g^r + g^l} g_0 \left[1 - \frac{(g_{\uparrow}^l g_{\downarrow}^r - g_{\downarrow}^l g_{\uparrow}^r)^2 / (g^r g^l g_{\uparrow} g_{\downarrow})}{1 + g_{\text{sf}} [(g_0 g_{\uparrow})^{-1} + (g_0 g_{\downarrow})^{-1}]} \right], \quad (24)$$

where g_{\uparrow}^l and g_{\downarrow}^l (g_{\uparrow}^r and g_{\downarrow}^r) are the spin-up and spin-down conductances of the left (right) junction in units of the quantum conductance e^2/h , and we have also introduced the combined conductances $g^l = g_{\uparrow}^l + g_{\downarrow}^l$, $g^r = g_{\uparrow}^r + g_{\downarrow}^r$, $g_{\uparrow} = g_{\uparrow}^l + g_{\uparrow}^r$, and $g_{\downarrow} = g_{\downarrow}^l + g_{\downarrow}^r$. The retarded correlation function g_0 (17) is the same as for an all-normal metal single-electron transistor. It renormalizes the conductance due to the Coulomb charging effects and depends on the induced electron number $C_g V_g / e$, the ratio of the charging energy to the temperature βE_c , and the coupling between the island and the reservoirs through the total conductance $g^r + g^l$. Through Eq. (24) we have thus reduced the problem of finding the conductance of ferromagnetic single-electron transistors to a well-studied problem of the renormalized conductance in all-normal metallic single-electron transistors. The second term in the bracket of (24) is a signature of the spin-accumulation and vanishes when $\tau_{\text{sf}} \rightarrow 0$ ($g_{\text{sf}} \rightarrow \infty$). The spin-accumulation causes a reduction of the conductance. The resulting conductance (24) can be understood in terms of the equivalent circuit shown in Fig. (2), which is identical to the high-temperature classical circuit for two spins,¹⁰ but now generalized to include Coulomb charging effects by renormalizing the conductances so that $g_{\uparrow}^l \rightarrow g_0 g_{\uparrow}^l$, $g_{\downarrow}^l \rightarrow g_0 g_{\downarrow}^l$, $g_{\uparrow}^r \rightarrow g_0 g_{\uparrow}^r$ and $g_{\downarrow}^r \rightarrow g_0 g_{\downarrow}^r$. The spin-flip conductance g_{sf} is not renormalized, since spin-flip processes do not change the number of electrons on the island and consequently not the charging energy. We will now use (24) to find the TMR ratio in F-N-F and F-F-F SET's.

IV. F-N-F MAGNETO-RESISTANCE

We first focus on F-N-F systems. The quantity of interest is the TMR $(g_P - g_{\text{AP}})/g_P$. When the magnetiza-

tions in the left and right leads are parallel, the tunnel conductances can be characterized as $g_s^l = g^l(1 + sP_l)/2$ and $g_s^r = g^r(1 + sP_r)/2$, where $s = +1$ ($s = -1$) denotes spin-up (spin-down) and $P_l = (g_{\uparrow}^l - g_{\downarrow}^l)/(g_{\uparrow}^l + g_{\downarrow}^l)$ and $P_r = (g_{\uparrow}^r - g_{\downarrow}^r)/(g_{\uparrow}^r + g_{\downarrow}^r)$ denote the polarizations of the left and right tunnel junctions, respectively. In the anti-parallel configuration the tunnel conductances are $g_s^l = g^l(1 + sP_l)/2$ and $g_s^r = g^r(1 - sP_r)/2$. The total conductances of each junction, $g^l = g_{\uparrow}^l + g_{\downarrow}^l$ and $g^r = g_{\uparrow}^r + g_{\downarrow}^r$, are thus independent of the magnetization configuration. Hence, the correlation function g_0 and consequently also the prefactor in (24) remain invariant on going from the parallel to the anti-parallel configuration. The magnetoresistive effect is thus solely due to the spin-accumulation which is contained in the second term of Eq. (24). The TMR of F-N-F systems is

$$\frac{g_P - g_{\text{AP}}}{g_P} = \frac{P_l P_r}{1 + \alpha_g^2 - \beta_g^2} (1 - \gamma_g^2), \quad (25)$$

where the parameter $\alpha_g^2 = 4g_{\text{sf}}/[g_0(g^l + g^r)]$ ($\alpha_g^2 \geq 0$) reduces the spin-accumulation and hence the TMR due to spin-flip relaxation, $\gamma_g = (g^l - g^r)/(g^l + g^r)$ ($0 \leq \gamma_g^2 \leq 1$) reduces the TMR due to the asymmetries in the conductances of the left and right junctions, and $\beta_g^2 = (P_l^2(1 + \gamma_g)^2 + P_r^2(1 - \gamma_g)^2 - 2P_l P_r(1 - \gamma_g^2))/4$ ($0 \leq \beta_g^2 \leq 1$). The TMR is proportional to the polarization of the left and the right tunnel conductance. This result (25) reduced to the high temperature or large bias result in Ref. 10 when $g_0 \rightarrow 1$.

The spin-flip relaxation time is governed by spin-orbit coupling and magnetic impurities at low temperatures. We assume that τ_{sf} is temperature-independent below the Coulomb charging energy (at very low temperatures this assumption can become invalid, but that is a regime beyond the investigation in this article). Firstly, since the correlation function g_0 can only decrease due to Coulomb charging effects the factor α_g appearing in (25) can only increase at low temperatures and *the TMR in the Coulomb blockade regime is equal to or smaller than the classical high-temperature TMR*. Secondly, when the spin-flip relaxation time is much larger than the dwell time, the TMR is independent of the charging energy, temperature and tunnel resistances for practically all temperatures since $\alpha_g \rightarrow 0$ except at extremely low temperatures. Thirdly, in this regime we expect the nonlinear response TMR to have a re-entrant behavior: The linear response TMR is identical to the TMR when the bias is much larger than the Coulomb charging energy which equals the classical high-temperature TMR. In the intermediate bias regime effects of the Coulomb charging energy appear^{7,9,10} in the sequential tunneling limit and an enhancement of the TMR has been demonstrated in Ref. 12 using the co-tunneling formalism. Finally (24) and (25) can be generalized to finite frequencies by letting $g_0 \rightarrow g_0(\omega)$ and $1/\tau_{\text{sf}} \rightarrow 1/\tau_{\text{sf}} + i\omega$ so that the AC response can be used to detect the spin-accumulation.¹⁰ Our general result (25) is consistent with the calculations

performed in the sequential tunneling regime.^{9–11}

V. F-F-F MAGNETO-RESISTANCE

The spin-flip relaxation time in a ferromagnet is typically much shorter than the transport dwell time.¹⁰ In the limit $\tau_{\text{sf}} \rightarrow 0$ the conductance of F-F-F systems reduces to¹³

$$g = \frac{g^l g^r}{g^l + g^r} g_0(\beta E_c, g^l + g^r, n_{\text{ex}}). \quad (26)$$

The TMR of F-F-F systems is defined as

$$\gamma = \frac{R_{\text{AP}} - R_{\text{P}}}{R_{\text{P}}} = \frac{g_{\text{P}} - g_{\text{AP}}}{g_{\text{AP}}}, \quad (27)$$

where $R_{\text{P}} = R_K/g_{\text{P}}$ ($R_{\text{AP}} = R_K/g_{\text{AP}}$) is the resistance of the FSET and g_{P} (g_{AP}) is the dimensionless conductance of the FSET when the magnetizations in the leads and the central island are parallel (anti-parallel). (Note that we use a slightly different definition for the relative tunneling magnetoresistance in F-N-F systems than in F-F-F systems in order to make our discussions coherent with previous work). For F-F-F systems it is assumed that the magnetizations in the leads remain parallel, and that only the magnetization direction of the central island changes by applying an external magnetic field.

The TMR is caused by the magnetization configuration dependence of the total junction conductances $g^l = g_{\uparrow}^l + g_{\downarrow}^l$ and $g^r = g_{\uparrow}^r + g_{\downarrow}^r$ and therefore also by the magnetization configuration dependence of the renormalization factor g_0 . The latter dependence gives rise to the enhancement of the TMR in the Coulomb blockade regime. The first term of the effective action (19) governing the renormalization of the conductance g_0 (17) only depends on the Coulomb charging energy of the device, and thus in the weak tunneling limit $g_0 \rightarrow 1$ and the dimensionless conductances and the TMR of the FSET reduce to the classical values. The second term in the action (19) is proportional to the sum of the dimensionless conductances of the tunnel junctions and consequently is magnetization configuration dependent.

In order to evaluate the TMR the conductances of the FSET in different alignments must be calculated. We characterize the polarizations of the junctions by the relative polarizations P and P' associated with the reservoirs and the island, respectively. In a simple two-band model with spin-independent tunneling probabilities the polarizations P and P' are directly related to the density of states in the reservoirs and the island. In general there is not such a one-to-one correspondence between the polarizations of the tunnel barriers and the polarizations of the density of states, but we can still characterize the spin-dependent asymmetries in the tunnel conductances by the parameters P and P' . We denote the dimensionless conductance of the left (right) junction averaged over the parallel and anti-parallel alignments by $\bar{g}^{l/r}$. In the

parallel configuration the spin-dependent conductances are

$$g_{\text{P},s}^{l/r} = \frac{1}{2} \bar{g}^{l/r} (1 + sP)(1 + sP'), \quad (28)$$

where $s = 1$ ($s = -1$) for spin-up (spin-down) electrons. Hence, the total classical dimensionless junction conductance is

$$g_{\text{P}}^{\text{cl}} = g_{\text{P}}^l + g_{\text{P}}^r = \bar{g}(1 + PP'), \quad (29)$$

where $\bar{g} = \bar{g}^l + \bar{g}^r$. Similarly, in the anti-parallel configuration the conductance is

$$g_{\text{AP},s}^{l/r} = \frac{1}{2} \bar{g}^{l/r} (1 + sP)(1 - sP'). \quad (30)$$

The total classical dimensionless junction conductance is thus

$$g_{\text{AP}}^{\text{cl}} = g_{\text{AP}}^l + g_{\text{AP}}^r = \bar{g}(1 - PP'). \quad (31)$$

In the high temperature limit, one can substitute the forms of g_{P}^{cl} and $g_{\text{AP}}^{\text{cl}}$ described above into the definition of the TMR (27), and obtain a simple expression $\gamma^{\text{cl}} = 2PP'/(1 - PP')$ for the classical TMR.

If the thermal energy is larger than the Coulomb charging energy, the path integrals can be evaluated semiclassically, and the lowest order quantum corrections to the TMR can be obtained. In this regime the conductance of the FSET is^{13,16}

$$g_0(\omega = 0) \simeq 1 - \frac{\beta E_c}{3} + (0.0667 + 0.0185g_{\eta}^{\text{cl}})(\beta E_c)^2 - \frac{8\pi^2}{\beta E_c} e^{-\pi^2/\beta E_c - g_{\eta}^{\text{cl}}/2} \cos(2\pi n_{\text{ex}}) \quad (32)$$

with $\eta = P(AP)$ for the parallel (anti-parallel) alignment. Substituting this expression into the definition of the TMR (27), we obtain

$$\gamma^{\text{se}} = \gamma^{\text{cl}} + \mu^{\text{se}} (1 + \gamma^{\text{cl}}) (\beta E_c)^2 + \delta\gamma(n_{\text{ex}}), \quad (33)$$

where $\mu^{\text{se}} = 0.02g_{\text{P}}^{\text{cl}}$, and the leading term of the gate voltage dependent part of the semiclassical TMR is

$$\delta\gamma(n_{\text{ex}}) = \frac{8\pi^2}{\beta E_c} (1 + \gamma^{\text{cl}}) e^{-\pi^2/\beta E_c} \times [e^{-g_{\text{AP}}^{\text{cl}}/2} - e^{-g_{\text{P}}^{\text{cl}}/2}] \cos(2\pi n_{\text{ex}}). \quad (34)$$

In Ref. 13, $n_{\text{ex}} = 0$ was considered and consequently the gate voltage dependent contribution (34) was disregarded. For large junction conductances or high temperatures, this term is exponentially suppressed and the influence of the gate voltage on the TMR is negligible. The gate voltage dependence part of the TMR is important at low temperatures. Although Eq. (34) is not accurate at very low temperatures, it can be used to estimate when $\delta\gamma(n_{\text{ex}})$ becomes relevant. For FSET's with positive polarizations $PP' > 0$ the conductance in the anti-parallel

configuration is smaller than the conductance in the parallel configuration $g_{\text{AP}}^{\text{cl}} < g_{\text{P}}^{\text{cl}}$, and the maximum TMR occurs when $n_{\text{ex}} = 0$. The TMR attains its minimum at $n_{\text{ex}} = 1/2$, and returns to the maximum at $n_{\text{ex}} = 1$ since the TMR is a periodic function of n_{ex} with period 1.

At low temperatures, an analytical expression of the TMR is not available. We will study the maximum and minimum value of the TMR at low temperatures via Monte-Carlo simulations. We use Ni leads with $P = 0.23$, and Co island with $P' = 0.35$ in all numerical calculations below.

A. TMR in the 'off'-state

In the absence of induced charges, the geometric phase factor $2\pi i k n_{\text{ex}}$ in the phase-phase correlation function (18) vanishes. The density matrix for each winding number k is then positive and the phase-phase correlation function can be computed directly using standard Monte Carlo simulation techniques. In our earlier work¹³ a large enhancement of the TMR, roughly 4 times larger than the classical TMR for $\bar{g} = 12$ and $E_c/k_B T = 40$, was found. In this section we will demonstrate that the TMR is further enhanced at even lower temperatures. We will also address whether even larger junction conductances increase or decrease the TMR ratio.

The enhancement of the TMR ratio can be clearly demonstrated in the co-tunneling regime, where the conductance can be calculated analytically.^{17,18} Since co-tunneling is a second order tunneling process and the first order sequential tunneling vanishes in the Coulomb blockade regime, the conductance of the FSET is proportional to the square of the classical conductance with a temperature dependent prefactor. Consequently the TMR (27) is enhanced to a value slightly larger than twice the classical TMR.⁸ A naive extension of this result is that beyond the sequential and co-tunneling regimes, the TMR is enhanced by a factor of n when the n -th order tunneling process dominates. However this picture is too simplified since contributions from different orders of tunneling have different temperature-dependent prefactors and one cannot in general specify the tunneling process that dominates for given junction resistances and temperature. In the strong tunneling regime perturbation theory breaks down and a number of higher order tunneling processes have to be taken into account simultaneously.

When the thermal energy is not much smaller than the charging energy, the semiclassical formula (33) can be modified to fit the TMR values by replacing the Coulomb charging energy by a renormalized charging energy. This substitution agrees well with the results of earlier investigations on the single-electron box.^{19,20} The renormalization scheme also predicts that the effective charging energy at sufficiently low temperature decreases for larger junction conductances due to the coupling to the leads.

This means that for a given temperature, the TMR in the strong tunneling regime is smaller for larger junction conductances. The role of strong tunneling is therefore *dual*.²¹ The constructive role is to increase the TMR by the inclusion of higher order tunneling processes at low temperature. The destructive role is caused by the renormalized charging energy to a smaller value $E_c^* \ll E_c$ so that the TMR is closer to the classical value at a given temperature.

When the thermal energy is much smaller than the Coulomb charging energy strong tunneling is manifested as an enhancement of the TMR by lowering the temperature. We compute the TMR values for two values of the classical total dimensionless conductance $\bar{g} = 12$ and $\bar{g} = 24$. For a given conductance \bar{g} and temperature the phase-phase correlation function (18) is computed via Monte-Carlo simulations for a sufficiently large number of discrete points in the imaginary time interval $[0, \beta]$. The phase-phase correlation functions after two succeeding N samplings are compared. If the curves are smooth and sufficiently close to each other, they predict roughly the same conductance after the analytical continuation of the Matsubara-Fourier components. The small difference determines the uncertainty of the final result, which is shown as the size of the data point in the figures below. When the deviation between the two phase-phase correlation functions is larger than the requested accuracy they are averaged to give the phase-phase correlation function for the case of $2N$ samplings, and a further simulation is performed for the next case of $2N$ samplings. This procedure is followed until the final result converges. Using the computed phase-phase correlation functions, the conductances of the TMR can be calculated via the Kubo's formula Eq. (17). The analytical continuation from the Matsubara frequency to the real frequency²² is performed via the Páde approximate.²³

The resulting TMR ratio in the absence of the gate voltage is shown in Fig. (3) for $\bar{g} = 12$ and 24. In both cases, the TMR values at low temperature are larger than the classical values, which shows that the TMR is enhanced by higher order tunneling processes in the Coulomb blockade regime. For $\bar{g} = 12$ at $k_B T/E_c = 0.01$, the TMR is enhanced by a factor of 8 compared to the classical value, which is almost twice as large as the value that we reported for $k_B T/E_c = 0.025$ in our previous work.¹³ For a given temperature, the TMR for $\bar{g} = 12$ is larger than the one for $\bar{g} = 24$ in the temperature region shown in Fig. (3), which means that the smearing of the Coulomb charging energy via higher order tunneling processes is stronger for larger junction conductances. Therefore for experimentally easily accessible temperatures, only moderate enhancement of the TMR for the FSET formed by tunnel junctions with extremely large conductances can be obtained. In order to experimentally observe large TMR ratios in SET's tunnel conductances of the order of 10 times the quantum conductance are recommended.

B. TMR in the 'on'-state

In the 'on'-state when $n_{\text{ex}} = C_g V_g / e = 1/2$ module 1, there is no Coulomb blockade effect even when the thermal energy is much smaller than the charging energy. In the sequential tunneling regime, the conductance of the FSET approaches half of the classical value when the temperature is lowered.²⁴ Beyond the sequential tunneling regime, the reduction of the conductance of the FSET by lowering the temperature is slow at relatively high temperatures, but at sufficiently low temperature the conductance attains values smaller than the conductance in the sequential tunneling regime.^{8,24} The conductance in the 'on'-state in the strong tunneling limit and its implication for the TMR will be investigated in this section.

In order to calculate the 'on'-state conductance in the strong tunneling regime by Monte-Carlo simulations, we reformulate the phase-phase correlation function as

$$\Gamma^{\text{on}}(\tau) = \Gamma^{\text{re}}(\tau) / Z^{\text{re}}, \quad (35)$$

where the function $\Gamma^{\text{re}}(\tau)$ is the mean value of the phase factor

$$\Gamma^{\text{re}}(\tau) = \frac{1}{Z^0} \sum_{k=-\infty}^{\infty} \int_{b_k} D\varphi e^{-S[\varphi]} \cos(\pi k) \times \cos[\varphi(\tau) - \varphi(0)], \quad (36)$$

with respect to the partition function Z^0 which is a direct summation of the density matrices for all winding numbers k , $Z^0 = \sum_{k=-\infty}^{\infty} \int_{b_k} D\varphi e^{-S[\varphi]}$ and the partition function at resonance is renormalized in a similar way,

$$Z^{\text{re}} = \frac{1}{Z^0} \sum_{k=-\infty}^{\infty} \int_{b_k} D\varphi e^{-S[\varphi]} \cos(\pi k). \quad (37)$$

In the above formulas, both the numerator and the denominator of the phase-phase correlation function $\Gamma^{\text{on}}(\tau)$ can be calculated with respect to the density matrices of the FSET with a geometric factor which equals one. The computation can therefore be performed in the same way as in the case of vanishing gate voltage. The complication is that we now have to calculate both $\Gamma^{\text{re}}(\tau)$ and Z^{re} instead of a single correlation function.

When the thermal energy is not much smaller than the charging energy, quantum fluctuations are small and the simulations are rather fast. Consequently the stable phase-phase correlation functions can be readily obtained using the approach described above. A typical result is shown in Fig. (4). At lower temperatures, the imaginary time β is longer, and more lattice points are needed to guarantee the convergence of the Trotter product. Moreover, the computed correlation functions for the same amount of sampling numbers turn out to fluctuate more strongly at lower temperatures as shown in Fig. (5). For

$\bar{g} = 12$ at $\beta E_c \geq 60$ and $\bar{g} = 24$ at $\beta E_c \geq 100$, the correlation functions computed via the approach of doubling the sampling number still fluctuate considerably within a realistic computing time. Therefore we do not attempt to perform simulations for temperatures lower than $\beta E_c = 100$. At the lowest temperature the correlation function at the points with large fluctuations is replaced by a best fit smooth function so that the analytical continuations in the Kubo's formula lead to a converging conductance of the FSET. The accuracy of this smoothing approach needs to be determined by long time Monte Carlo simulations in the future, since we cannot guarantee that the single data point at the lowest temperature is accurate within its size.

The TMR ratio as a function of the inverse dimensionless temperature is shown in Fig. (6). For $\bar{g} = 12$, the TMR first increases by lowering the temperature. The values are almost the same as the case of the vanishing gate voltage. Around $\beta E_c = 10$, the gate voltage dependent contributions to the TMR becomes important shifting the TMR value for $C_g V_g = e/2$ farther away from the value for $C_g V_g = 0$ at lower temperatures. Therefore while the TMR in the Coulomb blockade regime is enhanced further at lower temperatures, the TMR at resonance decreases rapidly to the values smaller than the classical TMR for $\beta E_c \geq 30$. Negative TMR values could possibly occur at even lower temperatures, which needs to be determined quantitatively by further investigations which at present cannot be performed due to the computing time required. For $\bar{g} = 24$, in contrast to the case of smaller junction conductances $\bar{g} = 12$, the TMR increases constantly and slowly by lowering the temperature down to a value of $0.01 E_c / k_B$. The curve more or less follows the tendency of the TMR in the case of vanishing gate voltage. This is understandable because the gate voltage dependent contributions to the TMR is proportional to the factor $\exp(-g_{\eta}^{\text{cl}}/2)$ as expressed in (34), which is smaller for larger \bar{g} , therefore the difference between the maximal TMR at $C_g V_g = 0$ and the minimal TMR at $C_g V_g = e/2$ determined by the gate voltage dependent contribution is also smaller, and the variation of the minimal TMR will change the direction at lower temperatures. In both cases, the difference between the maximal TMR and the minimal TMR is found to be larger at lower temperatures, therefore a wider variation range of the TMR can be achieved by tuning the gate voltage.

VI. CONCLUSION

We have derived a general formula for the linear conductance of single-electron transistors containing ferromagnetic elements. In F-N-F structures, the TMR is almost independent of the Coulomb charging energy and is only *reduced* at sufficiently low temperatures when the effective transport dwell time becomes shorter than

the spin-flip relaxation time of the normal-metallic island. In F-F-F systems, we have calculated the magneto-resistance as a function of gate voltages for a wide range of temperatures in the strong tunneling regime. In the 'off'-state, the magneto-resistance can be enhanced at low temperatures by higher order tunneling processes. However, higher order tunneling processes also reduce the effective charging energy, and slow down the rate of the enhancement of the magneto-resistance when the junction conductances are much larger than the quantum conductance. Consequently the largest magneto-resistance for a given temperature is obtained when the junction conductances are larger, but not very much larger than the quantum conductance. The magneto-resistance of the device in the 'on' state has a more complicated temperature dependence. Due to the contributions from the induced charges, the magneto-resistance increases slowly for very large junction conductances. In contrast to this, the magneto-resistance is reduced well below the classical magneto-resistance for not very large junction conductances at sufficiently low temperatures.

We are grateful to G. E. W. Bauer, K.-A. Chao, B. I. Halperin, J. Inoue, Yu. V. Nazarov, and A. A. Odintsov for stimulating discussions. A. B. would like to thank the Norwegian Research Council for financial support and X. H. W. acknowledges support from the Swedish Research Council.

-
- ¹ G. A. Prinz, *Science* **282**, 1660 (1998); P. M. Levy, *Sol. St. Phys.* **47**, 367 (1994); M. A.M. Gijs and G. E. W. Bauer, *Adv. Phys.* **46**, 285 (1997); P. M. Levy and S. F. Zhang, *Curr. Opin. Solid State Mater. Sci.* **4**, 223 (1999); A. Brataas, Yu. V. Nazarov and G. E. W. Bauer, *Phys. Rev. Lett.* **84**, 2481 (2000).
 - ² R. Meservey and P. M. Tedrow, *Phys. Rep.* **238**, 173 (1994).
 - ³ R. Fiederling, M. Kelm, G. Reuscher, W. Ossau, G. Schmidt, A. Waag and L. W. Molenkamp, *Nature* **402**, 787 (1999); Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura H. Ohne and D. D. Awschalom, *Nature* **402**, 790 (1999).
 - ⁴ K. Tsukagoshi, B. W. Alphenaar and H. Ago, *Nature* **401**, 572 (1999).
 - ⁵ J. S. Moodera, L. R. Kinder, T. M. Wong and R. Meservey, *Phys. Rev. Lett.* **74**, 3273 (1995); J. S. Moodera and J. Nassar and G. Mathon, *Annu. Rev. Mater. Sci.* **29**, 381 (1999).
 - ⁶ K. Ono, H. Shimada and Y. J. Ootuka, *J. Phys. Soc. Jpn.* **66**, 1261 (1997); L. F. Schelp, A. Fert, F. Fetta, P. Holody, S. F. Lee, J. L. Maurice, F. Petroff and A. Vaures, *Phys. Rev. B* **56**, R5747 (1997); S. Sankar, B. Dieny and A. E. Berkowitz, *J. Appl. Phys.* **81**, 5512 (1997).
 - ⁷ J. Barnas and A. Fert, *Phys. Rev. Lett.* **80**, 1058 (1998).
 - ⁸ S. Takahashi and S. Maekawa, *Phys. Rev. Lett.* **80**, 1758

- (1998).
- ⁹ J. Barnas and A. Fert, *Europhys. Lett.* **44**, 85 (1998).
- ¹⁰ A. Brataas, Yu. V. Nazarov, J. Inoue and G. E. W. Bauer, *Phys. Rev. B* **59**, 93 (1999); *Eur. Phys. J. B* **9**, 421 (1999).
- ¹¹ A. N. Korotkov and V. I. Safarov, *Phys. Rev. B* **59**, 89 (1999).
- ¹² H. Imamura, S. Takahashi and S. Maekawa, *Phys. Rev. B* **59**, 6017 (1999).
- ¹³ X. H. Wang and A. Brataas, *Phys. Rev. Lett.* **83**, 5138 (1999).
- ¹⁴ A. H. MacDonald, *cond-mat/9912392*.
- ¹⁵ G. Schön, and A. D. Zaikin, *Phys. Rep.* **198**, 237 (1990) and references therein.
- ¹⁶ X. H. Wang, *Phys. Rev. B* **55**, 12 868 (1997).
- ¹⁷ D. V. Averin and Yu. V. Nazarov, *Phys. Rev. Lett.* **65**, 2446 (1990).
- ¹⁸ D. V. Averin and K. K. Likharev in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991); G.-L. Ingold and Yu. V. Nazarov in *Single Charge Tunneling*, Vol. 294 of NATO Advanced Study Institute Series B, edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992).
- ¹⁹ M. Büttiker, *Phys. Rev. B* **36**, 3548 (1987).
- ²⁰ X. H. Wang, R. Egger and H. Grabert, *Europhys. Lett.* **38**, 545 (1997); and references therein.
- ²¹ We thank Drs. Korotkov and Nazarov for drawing our attention to this point.
- ²² G. D. Mahan, *Many-Particle Physics*, Chapter 3, (Plenum Press, New York, 1990).
- ²³ S. Chakravarty and J. Rudnick, *Phys. Rev. Lett.* **75**, 501 (1995); and J. E. Hirsh, in *Quantum Monte Carlo Methods in Equilibrium and Nonequilibrium Systems*, edited by M. Suzuki (Springer, Berlin Heidelberg, 1987).
- ²⁴ P. Joyez, V. Bouchiat, D. Esteve, C. Urbina and M. H. Devoret, *Phys. Rev. Lett.* **79**, 1349 (1997).

FIG. 1. The single-electron transistor. The central island is connected to the left and the right reservoirs by tunnel junctions. The electrostatic energy of the island can be tuned by a gate voltage capacitively coupled to the island.

FIG. 2. The equivalent circuit of the low-bias conductance of the FSET including the spin accumulation effect of the central island.

FIG. 3. Tunneling magneto-resistance for $C_g V_g = 0$ as a function of the inverse of the dimensionless temperatures in the strong tunneling regime for $\bar{g} = 12$ (filled dotted), and for $\bar{g} = 24$ (filled diamonds). The errors are smaller than the sizes of the symbols.

FIG. 4. Phase-phase correlation function as a function of the imaginary time for $\bar{g} = 12$, $\beta E_c = 20$ and $C_g V_g = e/2$. The dotted lines are the Monte-Carlo data for 10^6 samplings, each. The dot-dashed line is the average of the dotted lines, the dashed line is the Monte-Carlo data for next 2×10^6 samplings, and the solid line is the average of the dot-dashed line and the dashed line.

FIG. 5. Phase-phase correlation function as a function of the imaginary time for $\bar{g} = 12$, $\beta E_c = 60$ and $C_g V_g = e/2$. The dotted lines are the Monte-Carlo data for 10^6 samplings, each. The dot-dashed line is the average of the dotted lines, the dashed line is the Monte-Carlo data for the next 2×10^6 samplings, and the solid line is the average of the dot-dashed line and the dashed line. The smooth long dashed line in the figure is used to replace the fluctuating solid line to calculate the Matsubara-Fourier components of the correlation functions.

FIG. 6. Tunneling magneto-resistance for $C_g V_g = e/2$ as a function of the inverse of the dimensionless temperatures in the strong tunneling regime for $\bar{g} = 12$ (filled dotted), and for $\bar{g} = 24$ (filled diamonds). The errors are smaller than the sizes of the symbols except for the point at lowest temperature for each set of parameters, where the error estimation is not accurate.











